CONTRIBUTIONS FROM THE MINERALOGICAL DEPARTMENT OF COLUMBIA UNIVERSITY.
VOL. X. NO. 2.

Cystallographic Studies.

(*a*) The Morphology of Certain Organic Compounds.

(*b*) The Calcites of the New Jersey Trap Region.

(*c*) New Graphical Methods.

BY AUSTIN FLINT ROGERS.

A dissertation submitted to the Faculty of the School of Pure Science of Columbia University in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

NEW YORK CITY.
June, 1902.

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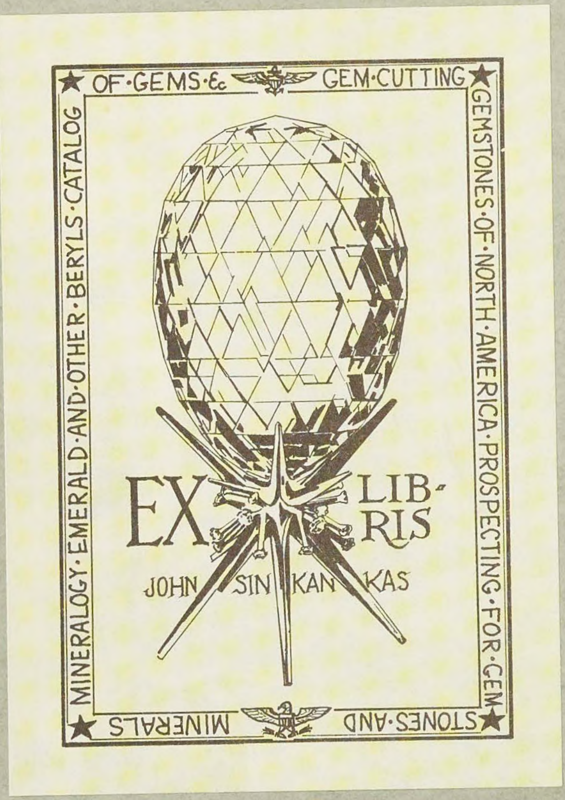
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THE MORPHOLOGY OF CERTAIN ORGANIC COMPOUNDS.*

BY AUSTIN F. ROGERS.

A crystallographic study of certain organic compounds furnishes the following notes. The potassium cyanate and mercuric oxycyanid were obtained from the chemical firm of Eimer & Amend. The other compounds are from the Laboratory of Organic Chemistry of Columbia University, and were kindly placed at the disposal of the writer by Prof. M. T. Bogert, of that department.

POTASSIUM CYANATE, KCNO.

Monoclinic:

$$a:b:c = 2.6177:1:1.2819:\beta = 86^\circ\ 11'.$$

Observed forms:

$$a\{100\}\ c\{001\}\ m\{110\}\ r\{101\}\ \{\bar{h}0l\}\ z\{112\}.$$

			Measured.		Calculated.	
cm	$(001:110)$	(12)	*94°	51′		
mm'	$(110:\bar{1}10)$	(12)	*41	54		
cr	$(001:101)$	(6)	*23	4		
ar	$(100:101)$	(4)	*53	7		
cr'	$(001:\bar{1}0\bar{1})$	(3)	126	51	126°	53′
mr	$(110:101)$	(3)	77	31	77	54
mr'	$(110:\bar{1}0\bar{1})$	(4)	102	22	102	6
cc	$(001:\mathit{001})$	(8)	9	44	9	42
$\bar{m}\underline{m}$	$(110:\mathit{110})$	(2)	96	9	96	12

Cleavage *r*, good; *m*, good; *a*, imperfect. Twins. Twinning plane *m*. Simple twins and cyclic groups.

Simple crystals exhibit the forms *m*, *c* and *r*. They are tabular in habit, being flattened parallel to one of the *m* faces, and this unequal development of the *m* faces gives them the appearance of triclinic crystals.

A second crop of crystals consisted almost entirely of twins of varying degree of complexity. Some are simple twins composed of two individuals (Fig. 1). Cyclic groups are more common and

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* From Ph.D. Thesis.

they made up of from 3 to 11 individuals on which the forms *a*, *c*, *m*, *r*, *z*, and an indetermined $\bar{h}0l$ are present (see Figs. 2–5). The cyclic arrangement is brought about by continued twinning parallel to two faces of the unit prism.

Twinned individuals on the exterior are always reduced to narrow bands and frequently such are introduced in the interior. This arrangement is due to successive twinning about the same face and

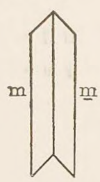


FIG. 1.

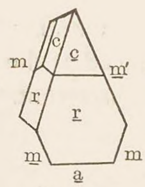


FIG. 2.

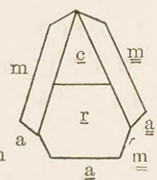


FIG. 3.

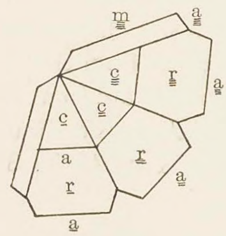


FIG. 4.

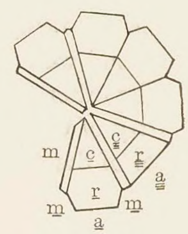
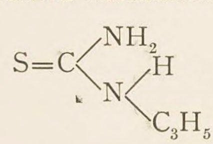


FIG. 5.

gives an appearance of twinning after two different laws. The cyclic twins simulate monoclinic domatic symmetry and not triclinic as do the simple crystals.

Brugnatelli * has described tetragonal crystals of potassium cyanate. They exhibit no cleavage, so are entirely distinct from our crystals.

ALLYL THIOCARBAMID.



(Prepared by W. Casamajor.)

Crystals prismatic in habit, usually singly terminated. Fig. 6 is an orthographic projection of such a crystal.

Monoclinic:

$$a : b : c = 0.9184 : 1 : 2.0044 ; \beta = 83°4'.$$

Observed forms:

$c\{001\}$ $m\{110\}$ $t\{012\}$ $q\{011\}$ $r\{\bar{1}01\}$ $o\{\bar{1}11\}$.

* *Ber. d. d. Chem. Ges.*, 27, 837, 1894. Abstr. *Zeit. f. Kryst. u. Min.*, 26, 628, 1896.

			Measured.		Calculated.	
mm'''	$(110 : 1\bar{1}0)$	(11)	*84°	43′		
cr	$(001 : \bar{1}01)$	(10)	*59	45		
ct	$(001 : 012)$	(3)	*44	51		
mm'	$(110 : \bar{1}10)$	(6)	95	17	95°	17′
$m'c$	$(\bar{1}10 : 001)$	(4)	73	22		
or	$(\bar{1}11 : \bar{1}01)$	(5)	41	0		
$m'r$	$(\bar{1}10 : \bar{1}01)$	(3)	63	17		
mm	$(110 : \mathit{110})$	(3)	33	34	33	16
$q\bar{t}$	$(011 : 012)$	(3)	17	41	18	28

Twins: (1) Twinning plane *c*. Have apparent hemimorphic orthorhombic symmetry. Fig. 7 (an orthographic projection with *c*, the twinning plane as the plane of projection).

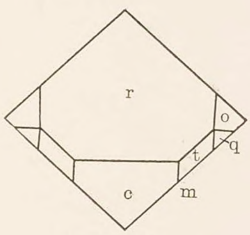


Fig. 6.

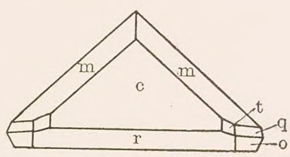


Fig. 7.

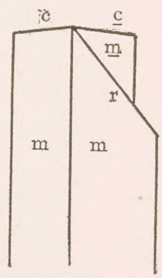


Fig. 8.

(2) Twinning plane *a*. Penetration twins resembling Carlsbad twins of orthoclase. There is an unequal development of the corresponding faces of the two individuals as indicated in Fig. 8.

Acetoxime, $(CH_3)_2C:NOH$.

(Prepared by A. Metzger.)

Acetoxime crystallizes in the hexagonal system, the crystals resembling the common type of quartz crystals (see Fig. 9).

Good crystals were obtained by allowing a specimen tube containing them to stand for some time when sublimed crystals were found on the sides of the tube. As the crystals deliquesce very rapidly in the air measurements were made with the crystals enclosed in a hollow cylinder * with calcium chlorid.

Hexagonal:

$$\dot{c} = 0.6541.$$

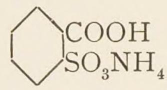
Observed forms:

$$m\{10\bar{1}0\} \qquad r\{10\bar{1}1\}.$$

* See Moses: "Characters of Crystals," p. 73, Fig. 74.

			Measured.	Calculated.
mr	$(10\bar{1}0 : 10\bar{1}1)$	(18)	52° 56′	
mm′	$(10\bar{1}0 : 01\bar{1}0)$	(15)	60 2	60° 0′

ACID AMMONIUM O-SULPHOBENZOATE,



The crystals of this salt are tabular, being flattened parallel to the basal pinnacoid and are rhombic in outline (Fig. 10).

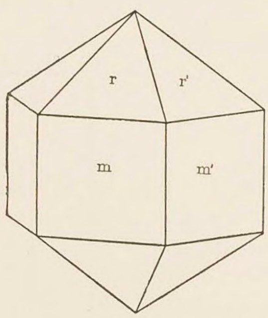


FIG. 9.

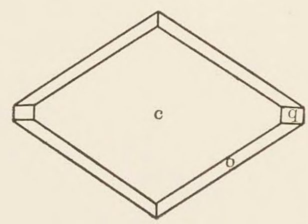


FIG. 10.

Orthorhombic:

$a : b : c = 0.6691 : 1.12158.$

Observed forms:

$c\{001\}$ $q\{011\}$ $o\{111\}$

			Measured.	Calculated.
co	(100 : 111)	(12)	*65° 25′	
oo‴	$(111 : \bar{1}\bar{1}1)$	(10)	*60 46	
oo^{iv}	$(111 : 11\bar{1})$	(3)	49 15	49° 10′
cq	(001 : 011)	(6)	50 28	50 29

Cleavage, perfect parallel to *c*.

MERCURIC OXYCYANID.

Crystals of two habits: (1) Obtuse sphenoidal with $o\{111\}$ dominant, (2) prismatic, with $a\{100\}$ dominant; $f\{201\}$, $o\{111\}$, and $c\{001\}$ are inconspicuous. Fig. 11, an orthographic projection, represents both habits.

Tetragonal. Scalenohedral class:

' = 0.4592.

Observed forms :

$a\{100\}$ $c\{001\}$ $f\{201\}$ $o\{111\}$ $o_1\{1\bar{1}1\}$.

			Measured.		Calculated.	
aa'	(100 : 010)	(12)	90°	2′	90°	0′
oo'	(111 : $\bar{1}\bar{1}1$)	(18)	*66°	0		
oo''	(111 : $11\bar{1}$)	(6)	134	44	134	42
af	(100 : 201)	(3)	47	30	47	36
oo_1'	(111 : $1\bar{1}1$)	(11)	45	19½	45	18

Benzyl Sulfocyanid, $C_6H_5CH_2SCN$.

Crystals somewhat tabular parallel to a. The merohedral development of the q faces may be accidental. Fig. 12 is an orthographic projection with 010 as the plane of projection.

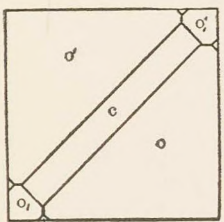


Fig. 11.

Monoclinic:

$a:b:c = 0.9019:1:0.7969;\ \beta = 83°\ 43'$.

Observed forms:

$a\{100\}$ $c\{001\}$ $m\{110\}$ $q\{011\}$.

			Measured.		Calculated.	
mm'''	(110 : $1\bar{1}0$)	(10)	*83°	45′		
mc	(110 : 001)	(4)	85	45	85°	19½′
ac	(100 : 001)	(10)	*83	43		
am	(100 : 110)	(7)	48	15	48	7½
cq	(001 : 011)	(5)	*38	23		
mq	(110 : 011)	(3)	58	37	58	58

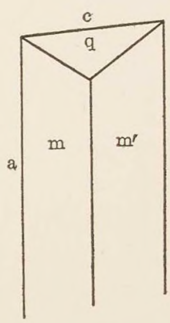


Fig. 12.

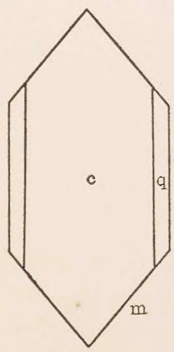


Fig. 13.

No cleavage was observed.

n-Propyl Dithiocarbamid,

$$S{=}C\begin{matrix}\diagup NH_2 \\ \diagdown S-C_3H_7\end{matrix}$$

(Prepared by M. T. Bogert.)

Crystals tabular parallel to basal pinacoid and elongated in the direction of the *a* axis (Fig. 13).

Monoclinic:

$$a:b:c = 0.8536:1:0.9447;\ \beta = 77^\circ\ 7'.$$

Observed forms:

$$c\{001\}\ a\{100\}\ m\{110\}\ t\{012\}\ q\{011\}.$$

			Measured.		Calculated.	
mm'''	$(110 \wedge \bar{1}10)$	(7)	*100°	28′		
cm	$(001 \wedge 110)$	(6)	*80	8		
cq	$(001 \wedge 011)$	(6)	*43	38½		
$c'm$	$(00\bar{1} \wedge 110)$	(3)	99	53½	99°	52′
qq'''	$(011 \wedge 01\bar{1})$	(4)	94	44	94	43
ct	$(011 \wedge 012)$	(3)	24	45	24	43

Cleavage perfect parallel to *c*, imperfect fibrous parallel to *a*.

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THE CRYSTALLOGRAPHY OF THE CALCITES OF THE NEW JERSEY TRAP REGION.

BY AUSTIN FLINT ROGERS.

Introduction.—The calcites of several different mineralogical provinces in America have been studied in some detail. Palache * has described the Lake Superior calcites. Hobbs † has given an account of the calcite crystals of the Upper Mississippi lead and zinc region. Farrington ‡ has presented a paper dealing with the well known calcites from the Joplin lead and zinc district.

The calcites occurring in the trap region of New Jersey, another distinct mineralogical province, have received but brief notice and it is my purpose in this paper to give a detailed account of such crystals as have come under my observation during the past year. The crystals have for the most part been collected by the writer. Some are in the Egleston Mineralogical Museum of Columbia University. Others have been furnished by Mr. Frederick Kato, of Jersey City, and by Mr. Wallace G. Levison, of Brooklyn, to both of whom I would express my sincere thanks.

Previous Work.—The only paper that has appeared on the crystallography of the New Jersey calcites is one by vom Rath in the Zeit. für Kryst. u. Min.§ Vom Rath describes five combinations

* Geol. Surv. Mich., Vol. VI., pp. 161–184. 1900.

† Zeit. f. Kryst. u. Min., Vol. XXV., 527.

‡ Publications Field Columbian Museum, Geol. Series. Vol. I., 232. 1900. The results of the writer's study of the Joplin calcites will appear in a forthcoming volume of the University Geological Survey of Kansas reports.

§ Vol. I., pp. 604–614; pl. XXV., ff. 2–6. 1877.

from Bergen Hill with sixteen forms, of which seven are doubtful.

Dana * has figured a calcite crystal from Bergen Hill with the forms ($-\frac{5}{4}R$ and $-\frac{7}{5}R\frac{3}{2}$), but no measurements are given.

Occurrence.—The calcites described in this paper are from the following localities, all in New Jersey: Bergen Hill, Jersey City, Weehawken, Edgewater, Fort Lee, Snake Hill, Upper Montclair, Great Notch and West Paterson.

They occur in veins in the trap or in the vesicular material at the contact between the trap and the Triassic sandstones and shales.

Habit.—The crystal habit varies from tabular to steep scalenohedral. Prismatic types are notably lacking. Crystals of tabular or low rhombohedral habit are common in the Palisade trap (intrusive) but have not been observed in the Watchung traps (extrusive). While crystals of steep rhombohedral or scalenohedral habit are present in both the Palisade and Watchung traps, they are relatively more abundant in the latter.

Description of Forms.

1. *Basal Pinacoid.*—This is a fairly common though usually subordinate form. On several combinations from Jersey City and Edgewater, it is the dominant form. At these localities the basal plane appears as a white opaque layer terminating the crystals in a similar manner to those of Andreasberg, Freiberg, and other European localities. On these tabular crystals parting parallel to the basal pinacoid is observed and is often very marked. The parting surface has a pearly luster.

The pinacoid is sometimes bright but it is easily affected by etching agents, and when at all exposed is corroded or etched.

2–3. *Prisms.*—The first and second order prisms are rare and subordinate forms. The prismatic habit is notably lacking in the New Jersey calcites.

4. *Positive Rhombohedrons.*—$m \cdot \{40\bar{4}1\}$ is a very common form. It usually has a bright surface.

5. $K \cdot \{50\bar{5}2\}$ occurs as very narrow faces truncating the obtuse polar edges of K: $\{21\bar{3}1\}$

6. $Q \cdot \{12.0.\overline{12}.5\}$. *new*, was observed as a subordinate form on some small crystals from Upper Montclair (combination 13).

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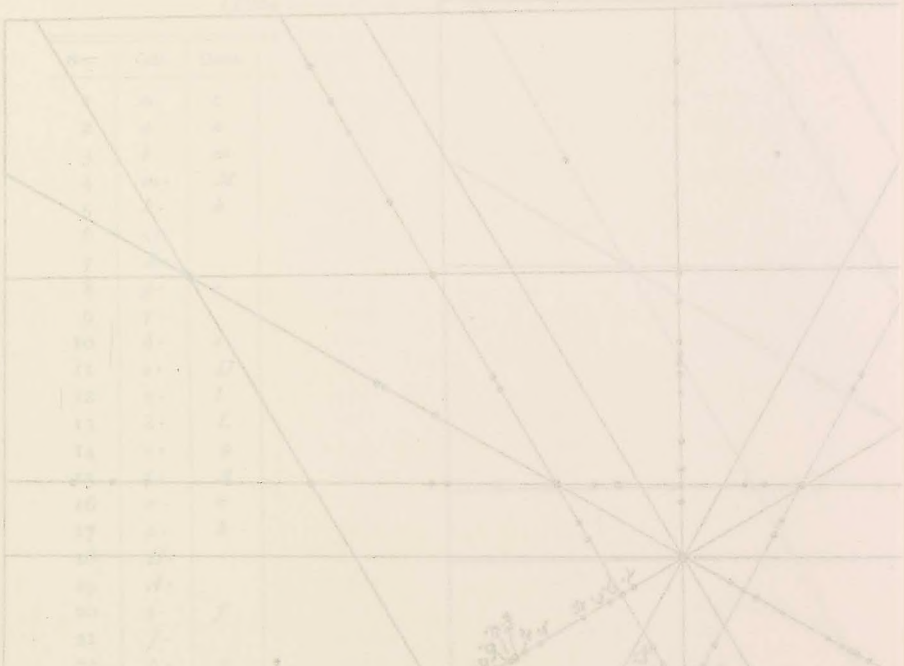
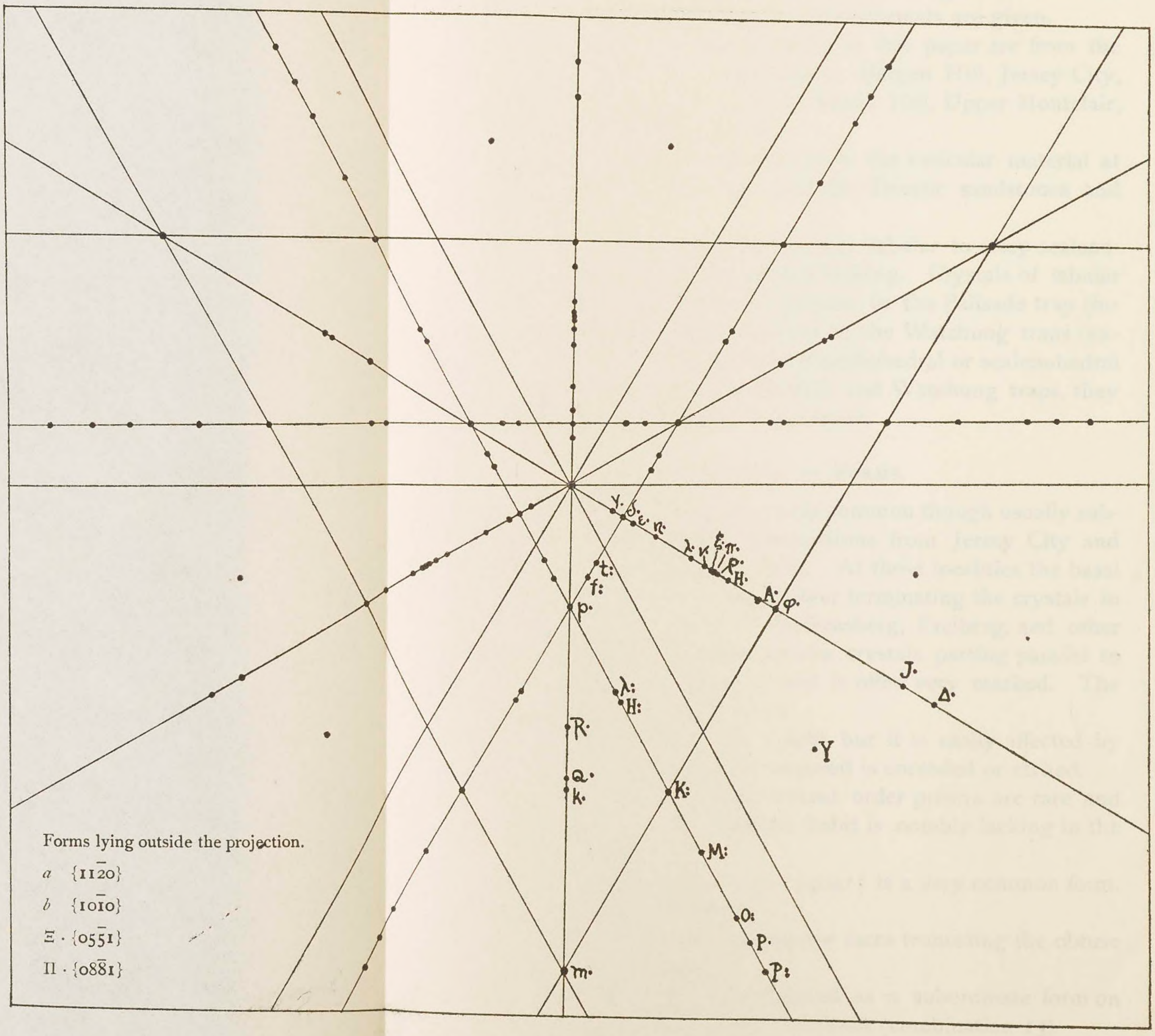
* "System of Mineralogy" (5th ed.), Fig. 552, p. 670. 1868.





Forms lying outside the projection.
a $\{11\bar{2}0\}$
b $\{10\bar{1}0\}$
$\Xi \cdot \{05\bar{5}1\}$
$\Pi \cdot \{08\bar{8}1\}$
γ
δ
ε
η
λ
κ
ξ
π
R
H
A
φ
J
Δ
Y
t
f
p
λ
H
R
Q
k
K
M
O
P
P
m

Table of Forms of the New Jersey Calcites.

(Plate I. is a gnomonic projection of the forms.)

No.	Gdt.	Dana.	Bravais-Miller.	Naumann.	Observer.
1	o	*c*	0001	$0R$	vom Rath.
2	*a*	*a*	$11\bar{2}0$	$\infty P2$	vom Rath.
3	*b*	*m*	$10\bar{1}0$	∞R	
4	*m* ·	*M*	$40\bar{4}1$	$+4R$	vom Rath.
5	*k* ·	*k*	$50\bar{5}2$	$+\frac{5}{2}R$	
6	*Q* ·		$12.0.\bar{12}.5$	$+\frac{12}{5}R$	*new.*
7	*R* ·		$20\bar{2}1$	$+2R$	
8	*p* ·	*r*	$10\bar{1}1$	$+R$	vom Rath.
9	γ ·		$02\bar{2}5$	$-\frac{2}{5}R$	
10	δ ·	*e*	$01\bar{1}2$	$-\frac{1}{2}R$	vom Rath.
11	ε ·	*D*	$03\bar{3}5$	$-\frac{3}{5}R$	
12	η ·	*l*	$04\bar{4}5$	$-\frac{4}{5}R$	
13	λ ·	*L*	$08\bar{8}7$	$-\frac{8}{7}R$	
14	ν ·	ϕ	$05\bar{5}4$	$-\frac{5}{4}R$	Dana.
15	ξ ·	*A*	$04\bar{4}3$	$-\frac{4}{3}R$	
16	π ·	π	$07\bar{7}5$	$-\frac{7}{5}R$	vom Rath only.
17	ρ ·	*h*	$03\bar{3}2$	$-\frac{3}{2}R$	
18	*H* ·		$0.18.\bar{18}.13$	$-\frac{18}{13}R$	*new.*
19	*A* ·		$09\bar{9}5$	$-\frac{9}{5}R$	
20	ϕ ·	*f*	$02\bar{2}1$	$-2R$	vom Rath.
21	*J* ·		$0.13.\bar{13}.4$	$-\frac{13}{4}R$	*new.*
22	Δ ·	χ	$07\bar{7}2$	$-\frac{7}{2}R$	
23	Ξ ·	*s*	$05\bar{5}1$	$-5R$	
24	Π ·	*d*	$08\bar{8}1$	$-8R$	
25	*t* :	*t*	$21\bar{3}4$	$+\frac{1}{4}R3$	
26	*f* :		$7.2.\bar{9}.11$	$+\frac{5}{11}R\frac{9}{5}$	vom Rath only.
27	λ :		$17.5.\bar{22}.2$	$+R\frac{11}{6}$	*new.*
28	*H* :	λ	$31\bar{4}2$	$+R2$	
29	*K* :	v	$21\bar{3}1$	$+R3$	vom Rath.
30	*M* :	(v_1)	$7.4.\bar{11}.3$	$+R\frac{11}{3}$	
31	*O* :		$8.5.\bar{13}.3$	$+R\frac{13}{3}$	
32	*P* ·		$17.11.\bar{28}.16$	$+R\frac{14}{3}$	
33	*P* :	*y*	$32\bar{5}1$	$+R5$	
34	*V* :	Ω_1	$6.5.\bar{11}.1$	$+R11$	
35	*Y*		$12.32.\bar{44}.13$	$-\frac{20}{13}R\frac{11}{5}$	vom Rath only.
			Doubtful Forms.		
			$12.6.\bar{18}.7$	$+\frac{6}{7}R3$	vom Rath only.
	R :		$10.7.\bar{17}.3$	$+R\frac{17}{3}$	" "
			$22.55.\bar{77}.36$	$-\frac{11}{12}R\frac{7}{3}$	" "
			$2.8.\bar{10}.5$	$-\frac{6}{5}R\frac{5}{3}$	" "
		μ	$17.74.\bar{91}.45$	$-\frac{19}{15}R\frac{91}{57}$	" "
		b	$7.35.\bar{42}.20$	$-\frac{7}{5}R\frac{3}{2}$	Dana only.
		ν	$11.62.\bar{73}.36$	$-\frac{17}{12}R\frac{53}{51}$	vom Rath only.

7. $R \cdot \{20\bar{2}1\}$. This form occurs on small crystals (combination 9) from Great Notch. Though subordinate it is well defined. The only previously recorded mention of the form is by Palache* on the Lake Superior calcites.

8. $p \cdot \{10\bar{1}1\}$.* The unit rhombohedron is the commonest of the positive rhombohedrons, but is much less common than some of the negative ones. On a group of Bergen Hill crystals in the Egleston Mineralogical Museum it is the only form present. These crystals which measure about an inch in diameter are similar to those from Poretta, Italy. Irby † states that it is extremely rare as a simple form.

9. *Negative Rhombohedrons.* The negative rhombohedrons outrank the positive ones in number and importance.

9. $\gamma \cdot \{02\bar{2}5\}$ was observed on crystals from Edgewater. Its faces are dull and rough.

10. $\delta \cdot \{01\bar{1}2\}$. This a common form with its characteristic striations parallel to the shorter diagonal of its faces but is always subordinate.

11. $\varepsilon \cdot \{0335\}$ was identified on a single minute crystal from Fort Lee (combination 24).

12. $\eta \cdot \{04\bar{4}5\}$. A crystal from West Paterson bears this form (combination 14).

13. $\lambda \cdot \{0887\}$ occurs at Great Notch and West Paterson.

14. $\nu \cdot \{05\bar{5}4\}$ is figured by Dana‡ on a Bergen Hill crystal. It was also identified by the writer on a crystal from Edgewater.

15. $\xi \cdot \{04\bar{4}3\}$ appears on several combinations from West Paterson.

16. π $\{07\bar{7}5\}$ is mentioned and figured by vom Roth as occuring on Bergen Hill calcites.

17. $\rho \cdot \{03\bar{3}2\}$ is a rather common form. On several crystals it is the dominant form. One crystal from Jersey City in the collection of Mr. Frederick Kato bears a remarkable resemblance to a cube. It is a prominent form on crystals from Edgewater where it appears in combination with $o\ \{001\}$. Some of the crystals have the appearance of a cube in combination with the octahedron. For further details see remarks under combination 10.

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* *Loc. cit.*

† On the Crystallography of Calcite. Inaug. Dis., Bonn, 1878. Abstr. in Zeit. f. Kryst. u. Min 3, 610, 1879.

‡ *Loc. cit.*

18. *H*·{0.18.$\bar{18}$.13} *new.* On the flat crystals from Edgewater, a new negative rhombohedron occurs to which this symbol is assigned.

19. *A*·{09$\bar{9}$5} occurs with the preceding form.

20. φ·{02$\bar{2}$1} is the most frequently occuring rhombohedron, and one to which many crystals owe their habit.

21. *J*·{0.13.$\bar{13}$.4} *new.* Narrow faces, truncating the obtuse polar edges of *P*·{17.11.$\bar{28}$.6}.

22. Δ·{07$\bar{7}$2} was observed but once, and then truncating the polar edges of *P*: {32$\bar{5}$1} on a small crystal from Upper Montclair.

23. Ξ·{05$\bar{5}$1} was identified by zonal relations on some crystals from Upper Montclair (combination 21).

24. π·{08$\bar{8}$1} was observed on Upper Montclair crystals, and is sometimes quite prominent. It also truncates the acute polar edges of *V*: {6.5.$\bar{11}$.1} of small crystals from Bergen Hill.

SCALENOHEDRONS OF THE PRINCIPAL ZONE.

This is a very prominent zone; all but two of the forms lie between {10$\bar{1}$1} and {11$\bar{2}$0}.

25. *t*: {21$\bar{3}$4} was observed on several crystals from the tunnel at Great Notch.

26. *f*: {7.2.$\bar{9}$.11} was observed by vom Rath.

27. λ: {17.5.$\bar{22}$.2}. *New.* A new form lying in a zone with δ·{01$\bar{1}$2}, *t*: {21$\bar{3}$4}, *p*·{10$\bar{1}$1} and {8.5.$\bar{13}$.3} is given the above indices. Although striated parallel to its intersection edges with *p*·{10$\bar{1}$1} it gave good reflections. It occurs on crystals from the tunnel at Great Notch and the combination is designated as No. 8.

28. *H*: {31$\bar{4}$2} is a prominent, sometimes the dominant, form, on crystals from Upper Montclair. Its faces are much striated, being in oscillatory combination with other scalenohedrons of the principal zone. It is associated either with *K*: {21$\bar{3}$1} or *P*: {32$\bar{5}$1}.

29. *K*: {21$\bar{3}$1} is a very common form and one to which many crystals owe their habit. The largest crystals found in the region (Bergen Hill and Edgewater) have this scalenohedron as a dominant form.

30. *M*: {7.4.$\bar{11}$.3} was observed but once and then as a prominent form on a minute crystal from Fort Lee (combination 24).

31. *O*: 8.5.$\bar{13}$.3}. A few crystals from the tunnel at Great-Notch bear this form.

32. $P\cdot$ $\{17.11.\overline{28}.6\}$, which has been recorded but once before (by Palache from Lake Superior), occurs as the dominant form on crystals from Great Notch.

33. P: $\{32\bar{5}1\}$ is quite common on the upper Montclair crystals. It is usually associated with K: $\{21\bar{3}1\}$ and the intersection edges of the faces of the two forms are not sharply defined. The faces are more brilliant than those of K:

34. V: $\{6.5.\overline{11}.1\}$ is the dominant form on crystals from two different localities, Bergen Hill (combination 27) and Snake Hill (combination 28).

35. *Other scalenohedrons.*—Vom Rath has described a number of other scalenohedrons but of these all but one $Y\{12.32.\overline{44}.13\}$ are doubtful.

Some crystals bear several other scalenohedrons, mostly negative ones, but from the measurements so far made, no indices can definitely be assigned to them.

Description of Combinations.

The various combinations are grouped under different types according to their dominant forms.

Type I.—Basal pinacoid dominant. Habit tabular. In the three combinations given the basal pinacoid is white and opaque and it is a plane of easy separation. The parting surfaces have a pearly luster.

Combination 1.—o, $\rho\cdot$ Edgewater. This combination grades into combination 11.

Measurements (contact goniometer). $o \wedge \rho\cdot$ $(0001:03\bar{3}2)$ (6) meas. 55°39′ calc. 55°37′.

2. o, $\varphi\cdot$ Jersey City. Small tabular crystals. $\varphi\cdot$ is identified by the fact that it truncates the polar edges of the cleavage rhombohedron.

3. o, $\varphi\cdot$, $A\cdot$, $H\cdot$, $\delta\cdot$, — ??, — ?? * Small tabular crystals. o and δ. are dull. The other planes are bright and give good reflections for the most part.

			Meas.	Calc.
$\delta\cdot \wedge H\cdot$	$(01\bar{1}2:0.18.\overline{18}.13)$	(6)	27°35′	27°32′
$\phi\cdot \wedge H\cdot$	$(02\bar{2}1:0.18.\overline{18}.13)$	(9)	9 10	9 20
$A\cdot \wedge H\cdot$	$(09\bar{9}5:0.18.\overline{18}.13)$	(3)	6 43	6 50
$p\cdot$ (cleav.) $\wedge H\cdot$	$(10\bar{1}1:0.18.\overline{18}.13)$	(3)	81 31	81 36½
$p\cdot$ (cleav.) $\wedge$ o	$(10\bar{1}1:0001)$	(3)	44 37	44 36½
$p\cdot$ (cleav.) $\wedge \phi\cdot$	$(10\bar{1}1:02\bar{2}1)$	( )	72 19½	72 16½
$\delta\cdot \wedge$ o	$(01\bar{1}2:0001)$	(2)	26 14½	26 15
$A\cdot \wedge \phi\cdot$	$(09\bar{9}5:02\bar{2}1)$	(3)	2 26	2 30
$A\cdot \wedge$ —??	$(09\bar{9}5:$? $)$	(3)	9 4	——

* In this and the following descriptions, ? is an undetermined rhombohedron, ? ? an undetermined scalenohedron, either positive or negative as indicated.

Type II. Prism dominant. No crystals observed.

Type III. Steep rhombohedron dominant (5 *R* or above). No determinable crystals of these types were observed.

Type IV. Middle rhombohedrons dominant (4 *R* to *R*).

4. $\varphi\cdot m\cdot$. Bergen Hill. Amber colored crystals. Forms identified by their relation to cleavage. Twinning lamellæ (tw. pl. $\delta\cdot\{01\bar{1}2\}$) are inserted in the crystals.

5. $\varphi\cdot$, *a*, + ??. Great Notch. Small colorless crystals of this combination are common at Great Notch.

6. $\varphi\cdot$, $\delta\cdot$, *a*, + ??. Upper Montclair.

7. ψ. Upper Montclair. Minute light amber-colored twinned crystals. The twinning plane is $p.\ \{10\bar{1}1\}$.

8. $\varphi\cdot$, $\delta\cdot$, t:, λ:, o:; *a*, $m\cdot$, *p*. Tunnel at Great Notch. Small colorless crystals. The faces in the zone [$\delta\cdot$, $p\cdot$, *a*] are striated, but give very good reflections. $\varphi\cdot$ has a very bright surface.

			Meas.	Calc.
$\lambda : \wedge \lambda :$	$(17.5.\overline{22}.2 : 22.\bar{5}.\overline{17}.2)$	(7)	25′ 1°44′	21°4
$\lambda : \wedge O :$	$(17.5.\overline{22}.2 : 8.5.\overline{13}.3)$	(3)	17 9	17 7
$t : \wedge t :$	$(21\bar{3}4 : 3\bar{1}\bar{2}4)$	(5)	20 25	20 36½
$\phi\cdot \wedge \phi\cdot$	$(02\bar{2}1 : \bar{2}021)$	(3)	78 48	78 51
$\phi\cdot \wedge \delta\cdot$	$(02\bar{2}1 : 01\bar{1}2)$	(3)	36 55	36 52

9. $\varphi\cdot$, $\rho\cdot$, $\lambda\cdot$, o, $p\cdot$, $R\cdot$, *b*, — ??, — ??. Great Notch. Small colorless crystals. The faces are more or less striated but gave fairly good reflections. $R\cdot$ is mentioned by Palache as occurring in the Lake Superior calcites. The basal pinacoid is smooth and bright. ρ is sometimes the terminal face to the exclusion of o.

			Meas.	Calc.
$\phi\cdot \wedge \rho\cdot$	$(02\bar{2}1 \wedge 03\bar{3}2)$	(2)	7°8′	7° 7′
$\rho\cdot \wedge \lambda\cdot$	$(03\bar{3}2 \wedge 08\bar{8}7)$	(2)	7 30	7 31½
$\varphi\cdot \wedge$ o	$(0221 \wedge 0001)$	(1)	63 17	63 7
$\lambda\cdot \wedge$ o	$(08\bar{8}7 \wedge 0001)$	(2)	48 45	48 25½
$R\cdot \wedge b$	$(20\bar{2}1 \wedge 10\bar{1}0)$	(4)	26 46	26 53
$p\cdot \wedge b$	$(10\bar{1}1 \wedge 10\bar{1}0)$	(2)	45 21	45 23½

Twin seams (tw. pl. $01\bar{1}2$) are present in a few crystals.

10. $\rho\cdot$ Bergen Hill, Weehawken. One crystal from the first locality with this form alone in the collection of Mr. Frederick Kato bears a remarkable resemblance to a cube. It is very symmetrical, measures about 6 mm. on a side and with the exception of another smaller one like it, is free on all sides being found perched on a tuft of byssolite. It is more like a cube than the

so-called cuboid of Haüy $(05\bar{5}4)$ the angle between adjacent faces being 91° 41′ instead of 84° 32½′. In the striations on the faces it simulates a crystal of the hextetrahedral class of the isometric system.

$\rho\cdot \wedge \rho\cdot$ $(03\bar{3}2 : \bar{3}30\bar{2})$ (6) meas. (contact goniometer) 88° 15′, calc. 88° 18′.

The Weehawken crystals of the same form are not so cube-like in appearance. They bear striations of a different sort from the Bergen Hill crystals. These striations were produced, or at least accentuated, by etching.

$\rho\cdot \wedge \rho\cdot$ $(03\bar{3}2 : \bar{3}032)$ (6) meas. (contact goniometer) 91° 45′, calc. 91° 42′.

11. $\rho\cdot$, o. Edgewater. Small nearly colorless crystals, simulating cubes with some of the angles acuminated by faces of octohedron. This combination grades into combination 1.

12. $\nu\cdot$ Edgewater. A single crystal consisting of this form alone was found at Edgewater.

$\nu\cdot \wedge \nu\cdot$ $(05\bar{5}4 : \bar{5}054)$ (6) meas. 84° 20′, calc. 84° 32½′.

13. $p\cdot, \varphi\cdot$, o, $m\cdot, b, Q\cdot, H\cdot, \xi\cdot$ + ??. Upper Montclair. Small colorless crystals intimately associated with heulandite. o is quite prominent on some crystals and has a bright luster.

			Meas.	Calc
o $\wedge\ \phi\cdot$	$(0001 : 02\bar{2}1)$	(2)	63°5½′	63°7′
$\phi\cdot \wedge b$	$(02\bar{2}1 : 10\bar{1}0)$	(2)	26 53½	26 53
$b \wedge m\cdot$	$(10\bar{1}0 : 40\bar{4}1)$	(2)	14 20	14 13
$m\cdot \wedge Q\cdot$	$(40\bar{4}1 : 12.0.\bar{12}.5)$	(2)	8 34	8 46
Q· $\wedge\ p\cdot$	$(12.0.\bar{12}.5 : 10\bar{1}1)$	(2)	22 28½	22 24½
o $\wedge\ H\cdot$	$(0001 : 0.18.\bar{18}.13)$	(3)	53 35	53 47
$\xi\cdot \wedge \phi\cdot$	$(04\bar{4}3 : 02\bar{2}1)$	(1)	10 13	10 22

14. $p\cdot, \varphi\cdot$, o, $m\cdot, \xi\cdot, \lambda\cdot, \eta., \delta\cdot$, —?? —??. West Paterson. Small colorless crystals similar to those of the last-mentioned combination.

			Meas.	Calc.
o $\wedge\ \phi\cdot$	$(0001 : 02\bar{2}1)$	(3)	63°5 ′	63°7 ′
o $\wedge\ \xi\cdot$	$(0001 : 04\bar{4}3)$	(3)	52 38	52 45
o $\wedge\ \lambda\cdot$	$(0001 : 08\bar{8}7)$	(3)	48 27	48 25 ½
o $\wedge\ \eta\cdot$	$(0001 : 04\bar{4}5)$	(3)	38 55	38 17
o $\wedge\ \delta\cdot$	$(0001 : 01\bar{1}2)$	(3)	26 20	26 15
$m\cdot \wedge p\cdot$	$(40\bar{4}1 : 10\bar{1}1)$	(3)	31 16	31 10 ½
$p\cdot \wedge$ o·	$(10\bar{1}1 : 0001)$	(3)	44 36	44 36 ½
$m\cdot \wedge \phi\cdot$	$(40\bar{4}1 : 0221)$	(3)	41 8	41 6

15. *p·* Bergen Hill. Dark grey crystals (about an inch in diameter) consists of this form alone. As a simple form *p·*, is quite rare. Poretta, Italy is a prominent locality for it.

Type V. Obtuse rhombohedron dominant.

16. *δ·*, *K*:, *P*:,—? Edgewater. Rather small crystals of tabular habit.

			Meas.	Calc.
$P:\wedge P:$	$(32\bar{5}1 \wedge 5\bar{2}\bar{3}1)$	(3)	45°5′	45°32′
$P:\wedge P:$	$(32\bar{5}1 \wedge 2\bar{3}\bar{5}1)$	(6)	29 0	29 15½
$K:\wedge K:$	$(21\bar{3}1 \wedge 3\bar{1}\bar{2}1)$	(6)	36 32	35 36

Type VI. Obtuse scalenohedron dominant.

16. *H*:, *b*, *δ*. Upper Montclair. Small slightly amber-colored crystals. *H*: is striated, being in oscillatory combination with other scalenohedrons of the principal zone.

			Meas.	Calc.
$H:\wedge H:$	$(3\bar{1}\bar{4}2 \wedge 4\bar{1}\bar{3}2)$	(3)	23°45′	24°10′
$b \wedge \delta\cdot$	$(10\bar{1}0 \wedge 01\bar{1}2)$	(2)	64°30′	63°45

17. *H*:, *P*:, Π·, *φ·* Upper Montclair. Small crystals similar to those of last mentioned combination.

			Meas.	Calc.
$H:\wedge H:$	$(3\bar{1}\bar{4}2 : 4\bar{1}\bar{3}\bar{3})$	(3)	24°00′	24°10′
$H:\wedge H:$	$(3142 : 34\bar{1}2)$	(2)	79 15	77 49
$\Pi\cdot \wedge \phi\cdot$	$(0\bar{8}81 : 02\bar{2}1)$	(2)	19 21	19 40
$\Pi\cdot \wedge$ p· (cleav)	$(08\bar{8}1 : 10\bar{1}1)$	(2)	52 56	52 36½

Type VII. Middle scalenohedron dominant.

18. *K*:, *k·* Edgewater. Rather small white crystals partially enclosed by a secondary growth of calcite. *k·* appears as very narrow faces truncating the obtuse polar edges of *K*: Measurements with contact goniometer, $K : K : (2\bar{1}31 : 3\bar{1}\bar{2}1)$ (4) meas. 35°25′, calc., 35°36′.

19. *K*: Edgewater. Large gray twinned crystal. Twin plane, o{0001}.

			Meas.	Calc.
$K:\wedge K:$	$(2\bar{1}\bar{3}1 : \bar{2}\bar{3}11)$	(3)	75°20′	75°22′
$K:\wedge K:$	$(2\bar{1}31 : 3\bar{1}\bar{2}1)$	(4)	35 35	35 36

20. *K*: Bergen Hill. Large crystals in the Egleston Mineralogical Museum associated with stilbite.

21. *K*:, *Ξ·*, *φ·* Upper Montclair. Small colorless crystals. The faces did not give good reflections. $K:\wedge K:$ Meas. (2) 36°43′, calc., 35°36′. *Ξ·* and *φ·* are identified by their zonal relations.

22. *K*:, *P*:, *m*·, ·, *p*· Upper Montclair. Small colorless or light amber crystals, very common at this locality. *P*: is bright and smooth while *K*: is rather dull. *K*: is nearly always dominant but sometimes *P*: and *K*: are almost equally developed. It is an interesting fact that the obtuse polar edges of *P*: and the acute polar edges of *K*: are invariably truncated, the one by *m*·, the other by *φ*·.

			Meas.	Calc.
$P: \wedge P:$	$(32\bar{5}1 : \bar{3}5\bar{2}1)$	(6)	71°3′	70°59′
$P: \wedge P:$	$(32\bar{5}1 : 5\bar{2}\bar{3}1)$	(3)	45 34	45 32
$P: \wedge P:$	$(32\bar{5}1 : 23\bar{5}\bar{1})$	(3)	29 19	29 15½
$K: \wedge K:$	$(21\bar{3}1 : \bar{2}3\bar{1}1)$	(6)	75 27	75 22
$K: \wedge K:$	$(21\bar{3}1 : 3\bar{1}\bar{2}1)$	(3)	35 32	35 36

m· and *φ*· were identified by their zonal relations.

23. *K*:, *φ*·, Upper Montclair. Small colorless twinned crystals associated with quartz crystals. The twinning plane is *p*.$\{10\bar{1}1\}$.

No measurements were possible on account of the dull surfaces of the faces but the forms were identified by their zonal and cleavage relations.

24. *M*:, *p*·, *φ*·, *ε*·. Fort Lee. Minute colorless twinned crystals tw. pl. *p*. $\{10\bar{1}1\}$.

			Meas.	Calc.
$M: \wedge M:$	$(7 \cdot 4 \cdot \overline{11} \cdot 3 : \bar{7} \cdot 11 \cdot \bar{4} \cdot 3)$	(4)	73°39′	73°40′
$M: \wedge M:$	$(7 \cdot 4 \cdot \overline{11} \cdot 3 : 11 \cdot \bar{4} \cdot \bar{7} \cdot 3)$	(3)	39 54	40 4
$m \cdot \wedge p \cdot$	$(40\bar{4}1 : 10\bar{1}1)$	(3)	31 9½	31 10½
$\phi \cdot \wedge p \cdot$	$(02\bar{2}1 : 10\bar{1}1)$	(3)	72 4	72 17
$\phi : \wedge \varepsilon \cdot$	$(02\bar{2}1 : 03\bar{3}5)$	(3)	32 52	32 30
$p : \wedge e \cdot$	$(10\bar{1}1 : 03\bar{3}5)$	(3)	104 56	104 47

25. *P*·, *J*·, —?. Great Notch. Small colorless crystals.

			Meas.	Calc.
$P \cdot \wedge P \cdot$	$(17.11.\overline{28}.6 : \overline{17}.28.\overline{11}.6)$	(8)	71°15′	71°12′
$P \cdot \wedge P \cdot$	$(17.11.\overline{28}.6 : 28.\overline{11}.\overline{17}.6)$	(13)	44 27	44 28

J· was identified by the fact that it truncates the acute polar edges of *P*·

26. *P*:, *Δ*·. Upper Montclair. Minute colorless crystals in cavities in prehnite.

P: $\wedge$ *P*: meas. (2) 45°46′. calc., 45°32′.

Δ· was recognized by the fact that it truncates the acute polar edges of *P*:

Type VIII. Acute scalenohedron dominant.

27. *V*:, Π·, o. Bergen Hill. Smaller colorless acicular crystals in the Egleston Mineralogical Museum.

V: is striated but gave good reflections. o is dull.

			Meas.	Calc.
V: ∧ *V*:	$(6.5.\overline{11}.1 \wedge \overline{6}.11.\overline{5}.1)$	(3)	65°34′	65°35½
V: ∧ *V*:	$(6.5.\overline{11}.1 \wedge 11.\overline{5}.\overline{6}.1)$	(3)	53 41	53 40

Π· was recognized by the fact that it truncates the acute polar edges of *V*:

28. *V*:, Π·. Snake Hill. Small nearly colorless crystals, capped by crystals of a second generation (ψ·, — ??, + ?) *V* is striated and gave poor reflections. Π· is very narrow.

V: ∧ *V*:	meas. (3)	65°41′	65°35½′

Twin Crystals.

Of the four twinning laws known for calcite, three are exemplified in the New Jersey trap calcites, namely, {0001}, $\{01\overline{1}2\}$ and $\{10\overline{1}\overline{1}\}$.

Crystals twinned according to the first law, where the twinning plane and composition is the basal pinacoid {0001} were observed in three different combinations all from one locality, Edgewater, large scalenohedron *K*: (combination 19.) tabular crystal o, and ρ· (comb. 1) and tabular crystal (comb. 3).

The second method of twinning (tw. pl. {0112}) is more common and was observed on crystals from Upper Montclair, West Paterson, Bergen Hill, and Snake Hill. The crystals from Snake Hill in the collection of Mr. W. G. Levison are the so-called "butterfly twins" and much resemble those from Guanajuto, Mexico, except that they are smaller.

Twin seams after this law occur on combination 4 from Bergen Hill and also on small crystals with undetermined negative scalenohedra from Upper Montclair. Twinning lamellæ caused by this method of twinning are frequent on cleavage surfaces.

Twinning after the third law (tw. pl. and comp. face $10\overline{1}1$) is exhibited by three different combinations:

1. Crystals from Upper Montclair. K:, ψ. (combination 23).
2. Crystals from Upper Montclair with the simple form φ.
3. Crystals from Fort Lee (combination 24).

The individual crystals have their vertical axes at right angles (89° 14′) and have one cleavage face in common.

These are, I believe, the first recorded occurrences of this twinning law for American calcites.

Parting.

In addition to the well-developed cleavage other planes of easy separation are frequently observed in the New Jersey calcites.

The well-known parting parallel to $\{01\bar{1}2\}$ due to secondary twinning is noticed and on some cleavage masses from Fort Lee this parting is more easily obtained than the cleavage itself. Several crystals from Edgewater broke with a surface of high lustre parallel to $\{01\bar{1}2\}$. Here there is apparently no connection with twinning. Such separation may be due to solution planes. That $01\bar{1}2$ is a plane of easy solution is shown by some crystals from Granby, Mo., studied by the writer. Etching agents had attacked the crystal producing cavities of various kinds but more especially flat cavities along the cleavage planes and along $(01\bar{1}2)$ sometimes penetrating quite a distance within the crystal.

In the flat tabular crystals with white opaque ends from Edgewater and Jersey City (combinations, 1, 2 and 3) perfect parting parallel to the basal pinacoid may easily be obtained within the white termination. The parting surfaces have a pearly luster. This method of parting is exhibited in crystals from Andreasberg, Freiberg, Kongsberg, Norway and Guauajuato, Mexico. It seems probable that the parting surfaces represent stages in the growth of the crystal though they are not marked by foreign inclusions as is the case in some other minerals.

Paragenesis.

Of the various minerals produced by contact metamorphism of the trap with the sandstones and shales calcite was one of the first to be formed, being preceded only by datolite and prehnite. The order of formation seems to have been about as follows: Datolite, prehnite, calcite, heulandite, apophyllite, analcite, natrolite, stilbite.

Several generations of calcite are observed. For instance at Great Notch a specimen of drusy calcite is distributed over other minerals including calcite. Phantom crystals and parallel growths of individuals of different habits are not uncommon.

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NEW GRAPHICAL METHODS IN CRYSTALLOGRAPHY.*

BY AUSTIN F. ROGERS.

(WITH PLATE I.)

VALUE OF GRAPHICAL METHODS.

To anyone who has worked in determinative crystallography the value of graphical methods is apparent. A graphical method for the solution of a crystallographic problem serves a twofold purpose:

1. It furnishes a check on calculation. The work must be checked in some way and graphical methods if sufficiently accurate are preferable to duplicate calculations.

2. It presents a picture of the mathematical operation involved and often enables one to see which of two courses to follow, more readily than do the formulæ used in calculation.

PREVIOUS USE OF GRAPHICAL METHODS.

Goldschmidt has treated the graphical solution of crystallographical problems quite elaborately in his "Ueber Projektion und graphische Krystallberechung." He solves the gnomonic projection completely for angles, elements and indices, but as developed by him it is so intimately connected with his polar symbols and elements that only those familiar with his system use it to any extent. In spite of some advantages of other notations for crystal faces and forms the Miller indices are decidedly the most compact and convenient.

Experience has shown that, all things considered, the most elegant method of representing the relative positions of the faces of a crystal, the zones which they form, and the kind of symmetry with which they are arranged, is by means of the stereographic projection.

The stereographic protractors recently designed and described

* From Ph.D. Thesis.

by Penfield * may be applied to the graphical solution of certain crystallographic problems, but these do not include the chief problems of crystallography, namely, the determination of the indices and the axial elements from the measured angles or conversely the determination of the angles from assumed indices and elements.

In the complete determination of a crystal there are three stages of calculation:

1. Calculation of indices from measured angles.

2. Calculation of axial elements from indices and measured angles.

3. Calculation of theoretical angles from axial elements and assumed indices.

It is the purpose in this paper to point out the fact that a graphic method suffices for the first stage, the determination of the indices and that it furnishes a check on subsequent calculation of theoretical angles from the indices and axial elements. Hence we have two stages of the calculation instead of three and the proper importance is given to the graphical determination.

Calculation may not be dispensed with, but except in work for record and comparison the graphic method, as outlined in the following pages, serves for the complete determination of the morphology of a crystal without any calculation whatever.

Graphical Determination of Indices.

A graphical method, perhaps the one best adapted to general use, for the determination of the indices and elements has been elaborated in another article in this issue of the Quarterly.† In general the construction is limited to one quadrant and so does not interfere with the rest of the projection.

But the disadvantage of this method is that it involves extra construction. It is important for the beginner to follow the construction, but after this is understood the protractor about to be described may be employed.

* *Amer. Jour. Sci.* (4), 11; 1–24, 115–143. 1901.

† pp. 11–22.

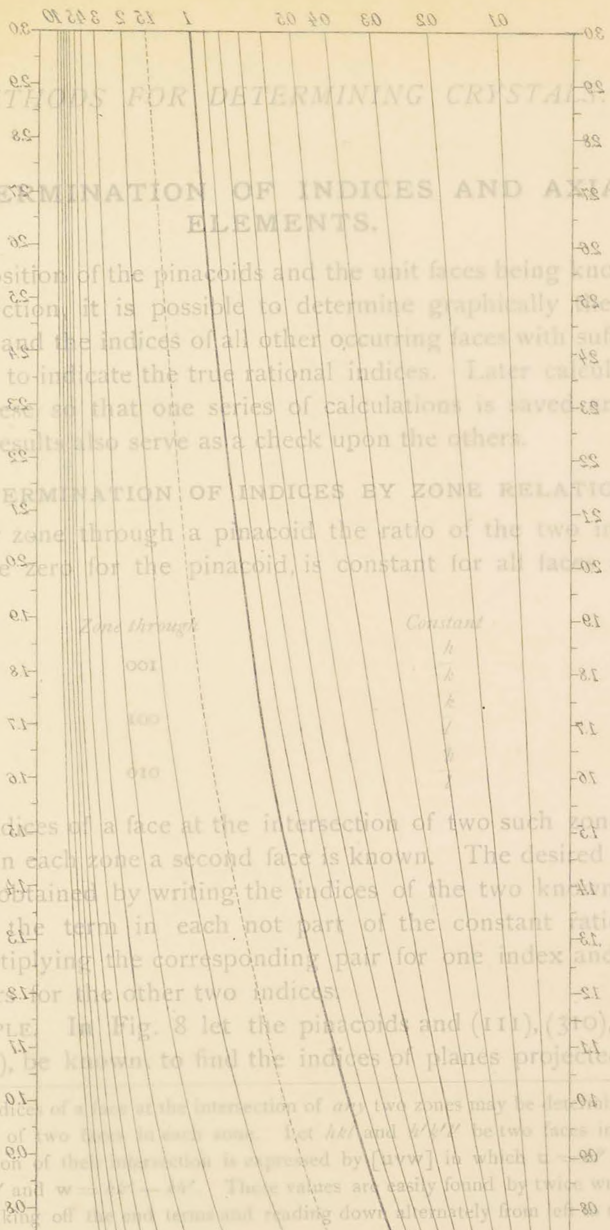

PLATE I.

Protractor for Determining Indices. Designed by Austin F. Rogers.

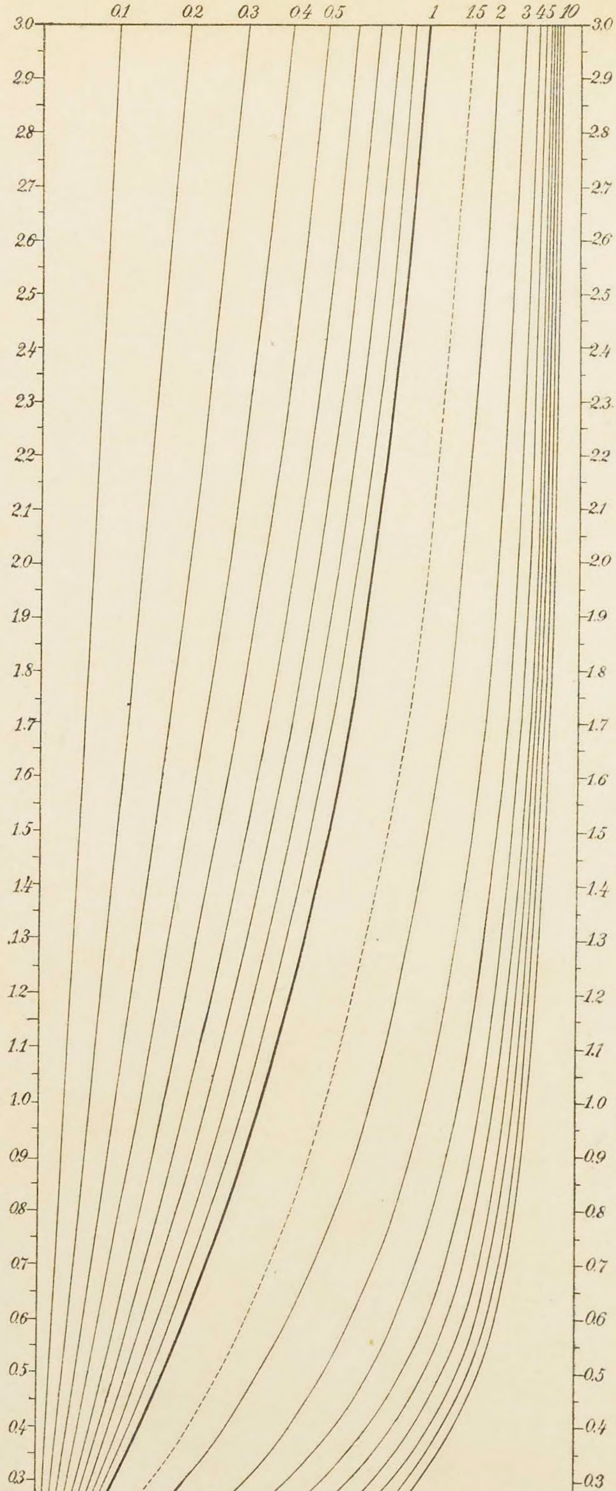

PLATE I.

Protractor for Determining Indices. Designed by Austin F. Rogers.

DETERMINATION OF INDICES AND AXIAL ELEMENTS.

The position of the pinacoids and the unit faces being known in the projection, it is possible to determine graphically the axial elements and the indices of all other occurring faces with sufficient accuracy to indicate the true rational indices. Later calculations check these so that one series of calculations is saved and the graphic results also serve as a check upon the others.

A. DETERMINATION OF INDICES BY ZONE RELATIONS.*

In any zone through a pinacoid the ratio of the two indices, which are zero for the pinacoid, is constant for all faces of the zone:

Zone through	*Constant*
001	$\frac{h}{k}$
100	$\frac{k}{l}$
010	$\frac{h}{l}$

The indices of a face at the intersection of two such zones will result if in each zone a second face is known. The desired values may be obtained by writing the indices of the two known faces omitting the term in each not part of the constant ratio, and then multiplying the corresponding pair for one index and diagonal pairs for the other two indices.

EXAMPLE. In Fig. 8 let the pinacoids and (111), (310), (305), and (130), be known, to find the indices of planes projected at *a*,

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* The indices of a face at the intersection of *any* two zones may be determined from the indices of two faces in each zone. Let *hkl* and *h'k'l'* be two faces in a zone. The direction of their intersection is expressed by [uvw] in which u $= kl' - lk'$, v $= lh' - hl'$ and w $= hk' - kh'$. These values are easily found by twice writing the indices, striking off the end terms and reading down alternately from left to right and from right to left, thus:

$$\begin{array}{c|cccc|c} h & k & l & h & k & l \\ & & \times & \times & \times & \\ h' & k' & l' & h' & k' & l' \end{array}$$

The values of [u'v'w'], the symbol of the second zone, are found in the same manner.

The values of (pqr), the desired indices, result from an exactly similar cross multiplication of [uvw] and [u'v'w'].

If a face lies in a zone [uvw] its indices must satisfy the equation pu $+ q$v $+ r$w $= 0$.

b, *c*, *d*, *e*, *f*, *g*, *h*, *i*, *k*. All calculations are either like *h* in zones [010 305] and [100 111],or like *i* in zones [001 130] and [100

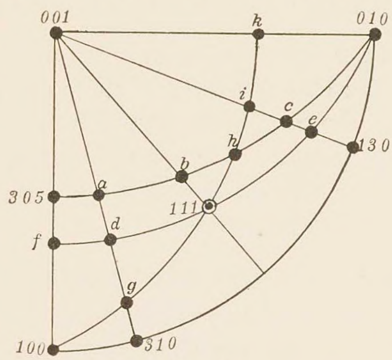

FIG. 8.

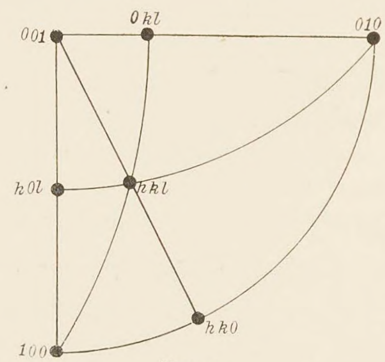

FIG. 9.

111]

$$\begin{array}{ccc} 3 & - & 5 \\ -1 & 1 & \\ \hline 3 & 5 & 5 \end{array} \quad h = (355),$$

$$\begin{array}{ccc} 1 & 3 & - \\ - & 1 & 1 \\ \hline 1 & 3 & 3 \end{array} \quad i = (133).$$

Similiarly $a = (315)$, $b = (335)$, $c = (395)$, $d = (313)$, $e = (131)$, $f = (101)$, $g = (311)$ and $k = (011)$. By considering zones through *a*, *b*, *c*, *d*, etc., other indices result.

B. GRAPHIC DETERMINATION OF INDICES AND AXIAL ELEMENTS.

NOTE.—For any face *hkl* the zone circles through 001, 100 and 010, Fig. 9, are found by protractor IV. and the coördinate angles * of the corresponding *hk*0, 0*kl* and *h*0*l* planes are found by protractor I.

Principle of Graphic Determination by *hk*0, 0*kl* and *h*0*l* Planes.

If *XY*, Fig. 10, is a quadrant of any zone, *O*, the center, and *P* and *P′*, two faces of that zone, then the intercepts *RT* and *RT′* made on any line *RW* normal to *OX* by the face-normals *OP*, *OP′* are in the ratio of the tangents of the angles *TOR* and *T′OR* respectively, *i. e.*,

$$\frac{RT'}{RT} = \frac{\tan T'OR}{\tan TOR}.$$

* The values of ρ for 0*kl* and *h*0*l* are respectively η_0 and ξ_0 of corresponding *hkl* in Goldschmidt's "Krystallographische Winkeltabellen."

If OX and OY are axes RT and RT′ are in the ratio of the Miller indices, for:

Let the dotted lines SM and SM' be drawn parallel to the crystal faces of which P and P' respectively are the poles.

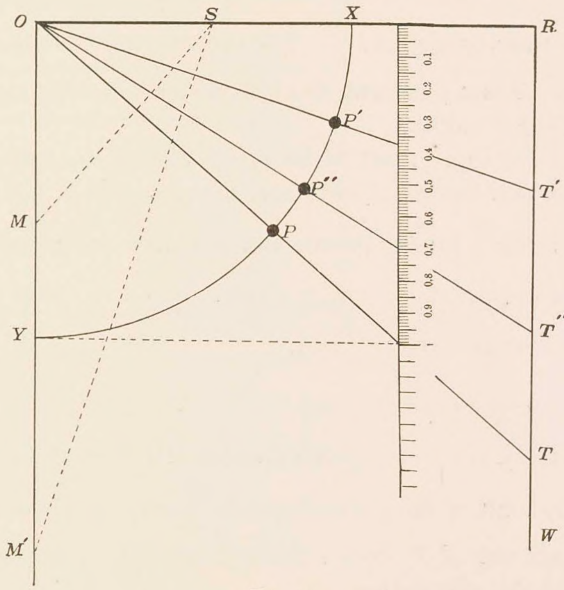

FIG. 10.

Then the angles $OMS = TOR$ and $OM'S = T'OR$, but

$$\frac{\tan T'OR}{\tan TOR} = \frac{\tan OM'S}{\tan OMS} = \frac{OM}{OM'};$$

hence
$$\frac{RT'}{RT} = \frac{OM}{OM'}.$$

That is, if the intercepts of the faces corresponding to P and P' on OX are made equal, their intercepts RT and RT' on RW are as the reciprocals of their intercepts OM and OM' on Y, that is, in the ratio of the indices.

This ratio is graphically obtained as follows:

RW is so drawn that RT is unity of some decimal scale,* then RT' and RT'', determined by radii through planes P' and P'', are given directly on the scale in terms of RT. In the drawing $RT' = 0.375$ and $RT'' = 0.7$.

* The radius of the circle is a convenient unit.

The relations are simplest when P is a unit plane. Then the readings on the scale line are the ratios $\frac{h}{k}$, $\frac{k}{l}$, or $\frac{h}{l}$.

Isometric, Tetragonal and Orthorhombic Crystals.

Determination of the ratio $\frac{h}{k}$. The construction for the *hk*o planes is made in the first quadrant, the face normals having already been laid off at the φ angles.

EXAMPLE, in Fig. 11 let M be 110. Taking the radius as unity of the scale and finding T as before, the scale line RT is drawn. Then the ratios $\frac{h}{k}$ for the prisms and pyramids are as follows:

Prism m and pyramids o, f, g, z and τ, $\frac{h}{k} = 1$.

Prism h and pyramid t, $\frac{h}{k} = \frac{1}{2}$.

Prism k and pyramid s, $\frac{h}{k} = \frac{1}{3}$.

If the intersection of a given face-normal with the scale line is beyond the limit of the paper the ratio $\frac{k}{h}$ may be obtained from a second scale line R_1T_1 drawn through a point T_1 the intersection of BT_1 and the unit radius.

EXAMPLE prism λ, $\frac{k}{h} = \frac{1}{2}$; $\frac{h}{k} = 2$. Prism N, $\frac{k}{h} = \frac{1}{3}$; $\frac{h}{k} = 3$.

Determination of ratios $\frac{k}{l}$ *and* $\frac{h}{l}$. The constructions for the *okl* and *hol* planes are conveniently made in the fourth quadrant, the ρ angles being laid off from A'. The radii CT' and CT'' for the unit domes determine the scale lines for the ratio $\frac{k}{l}$ and $\frac{h}{l}$ respectively.

EXAMPLE, Fig. 11,

okl planes, v, $\frac{k}{l} = \frac{1}{3}$ o, $\frac{k}{l} = 1$,

hol planes, e, $\frac{h}{l} = \frac{1}{3}$ d, $\frac{h}{l} = \frac{1}{2}$.

The *hkl* planes.—Either find $\frac{k}{l}$ and $\frac{h}{l}$ by a graphic determina-

tion of the corresponding *okl* and *hol* or if pyramids in the same zone are compared, obtain the indices directly from ρ.

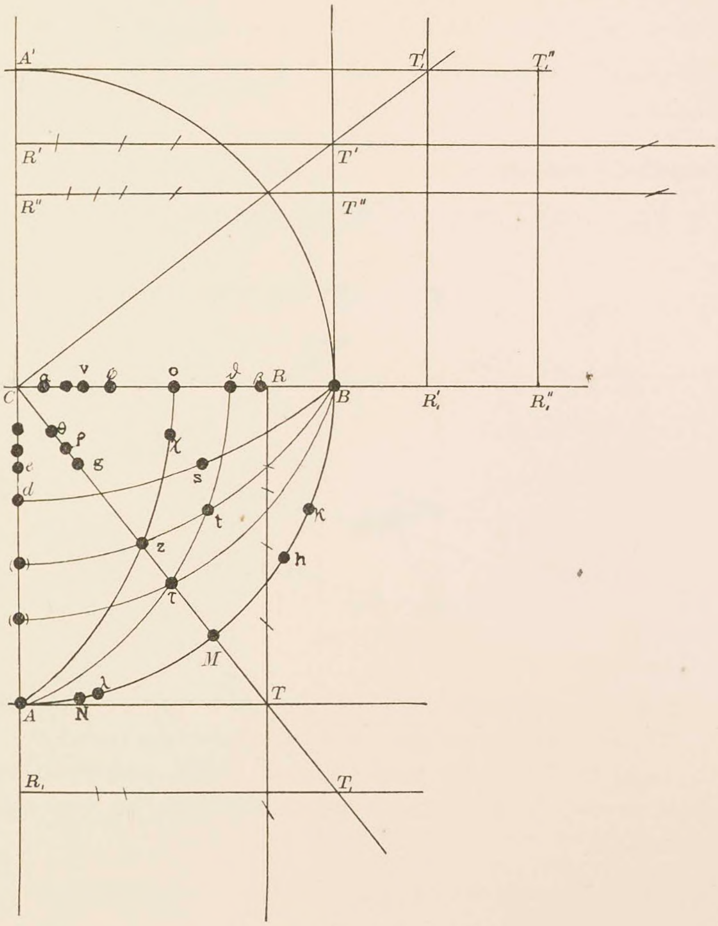

FIG. 11.

EXAMPLE. The *hkl* plane *s* is 132 for ρ of the corresponding *okl* gives $\frac{k}{l} = \frac{3}{2}$, and from *hol* gives $\frac{h}{l} = \frac{1}{2}$, or in comparison with known pyramid *hkl* (131) gives $\frac{l}{l'} = \frac{2}{1}$.

As in *hko* planes, a second scale gives may be drawn from T_1' or T_1'' parallel to $A'C$ for determining the ratio $\frac{l}{k}$ or $\frac{l}{h}$.

Determination of Axial Elements. Orthorhombic. Taking $b = 1 = CA$, then $AT = a$ on the same scale. Similarly, if $b = 1 = CA$, then $c = A'T_1'$. Similarly, if $b = 1 = CB$, then $BT'' = \frac{a}{c}$.

Tetragonal. c as in orthorhombic. If ρ of 111 is laid off $A'T_1' = 1.4142\,c$.

Hexagonal Crystals.

For any face $hkil$ the zone circles through 0001, $10\bar{1}0$, and

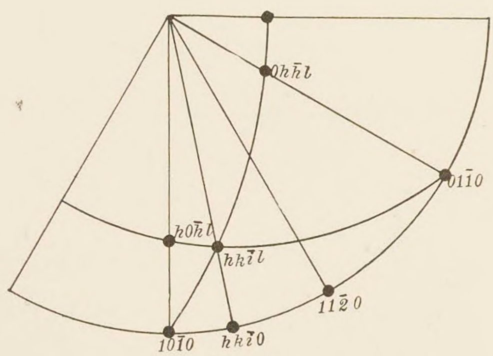

FIG 12.

$0\bar{1}10$, Fig. 12, and the coördinate angles of the corresponding $hk\bar{i}0$, $oh\bar{h}l$, and $ho\bar{h}l$ planes are found by protractors I. and IV.

All faces in the zones through 0001, $10\bar{1}0$, and $01\bar{1}0$ have constant ratios for the indices which are zero in the planes mentioned.

For a zone through 00(0)1, $\frac{h}{k}$ is constant for all faces.

For a zone through $10(\bar{1})0$, $\frac{k}{l}$ is constant for all faces.

For a zone through $01(\bar{1})0$, $\frac{h}{l}$ is constant for all faces.

The method of finding these ratios is essentially as described on page 12, but it is convenient to omit the third index.

EXAMPLE. $hk\bar{i}l$ lies in the zone $10\bar{1}0 : 02\bar{2}1$ and also in the zone $0\bar{1}10 : 11\bar{2}1$. By the first zone $\frac{k}{l} = \frac{2}{1}$, by the second zone $\frac{h}{l} = \frac{1}{1}$. Writing the three members hkl, omitting $\bar{i}$

```
— 2 1
  ×|
1 — 1
─────
1 2 1
```

$hkl = 121 \qquad hk\bar{i}l = 12\bar{3}1.$

Determination of ratio $\frac{h}{k}$. From the projections of $01\bar{1}0$ and $10\bar{1}0$, R and R', Fig. 13, draw lines RT and $R'T$ parallel to CR' and CR respectively. Intersections of radii through $hk\bar{i}l$ or $hki0$ on RT will give the ratio $\frac{h}{k}$ directly as $RT = CR =$ unity of scale. Similiarly radii on $R'T$ will give $\frac{k}{h}$.

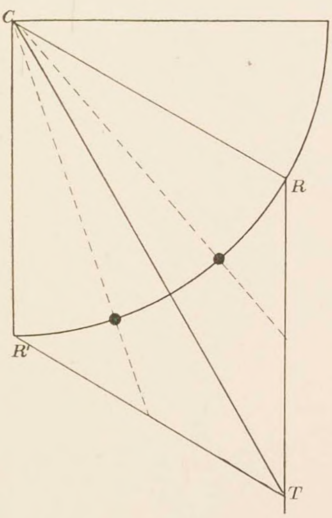

FIG. 13.

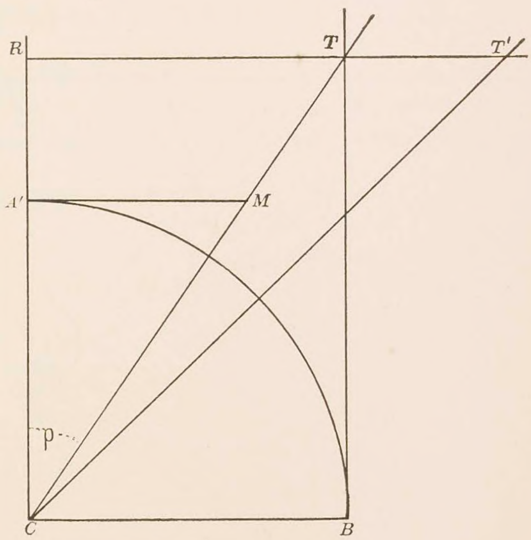

FIG. 14.

Determination of ratio $\frac{k}{l}$ or $\frac{h}{l}$. Lay off the angles from A'. If CB, Fig. 14, is the unit of the scale, the intersection T of the tangent and the radius through the unit face will give $RT = 1$ and other intercepts on the scale line such as RT' will be $\frac{h}{l}$. This is true in any vertical zone if ρ of the plane in that zone in which $\frac{h}{l}$ or $\frac{k}{l}$ is used to determine the position of the scale line RT.

Determination of the Axial Elements. If ρ of $11\bar{2}1$ is used and

$CA' = a$, then the tangent $A'M = \dot{c}$, measured on the scale. See Fig. 14. If ρ of 1011 is used, $A'M \times 0.866 = \dot{c}$.

Monoclinic System.

Determination of Ratio $\frac{h}{k}$. Prisms are determined as in the orthorhombic system. For pyramids the corresponding prisms are found by protractor IV. on zones through 001 which are no longer radial.

Determination of Ratio $\frac{h}{l}$. In * the fourth quadrant draw radii at angles ρ to CA for planes 001, 101, hol, etc., Fig. 15. Draw a line parallel to CB such that $RT =$ unit of scale line. Then $RT' = \frac{h}{l}$, $RT'' = \frac{h'}{l'}$, $R\bar{T}, = \bar{1}$, $R\bar{T}' = \frac{h}{l}$ on the same scale.

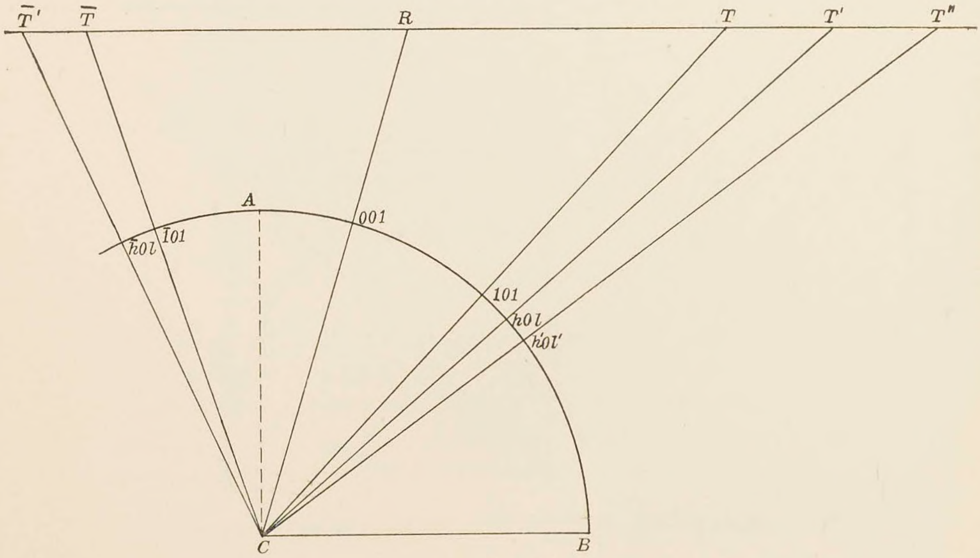

FIG. 15.

Determination of Ratio $\frac{k}{l}$.—Graphically find ρ of an imaginary

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* Proof. Let XY (Fig. 18) be face 101, $X'Y$, face hol, OY normal to OR, the axis $\dot{a}$ and OC, the axis $\dot{c}$. The triangles TOR and XOY, are similar, hence $\frac{RT}{OR} = \frac{OX}{OY} = \frac{c}{a}$. Similarly $\frac{RT'}{OR} = \frac{h}{l}\frac{c}{a}$. Hence $\frac{RT'}{OR} \times \frac{OR}{RT} = \frac{h}{l}\frac{c}{a} \times \frac{a}{c}$. $\therefore \frac{RT'}{RT} = \frac{h}{l}$. When $RT = 1$, $RT' = \frac{h}{l}$.

corresponding face M on ZB, Fig. 16. Then determine as in the orthorhombic system.

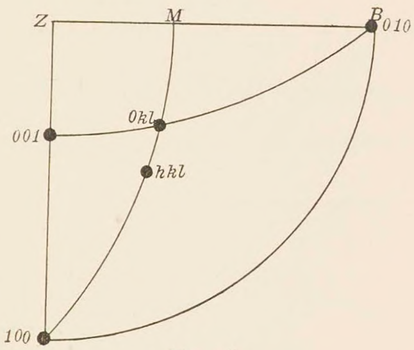

FIG. 16.

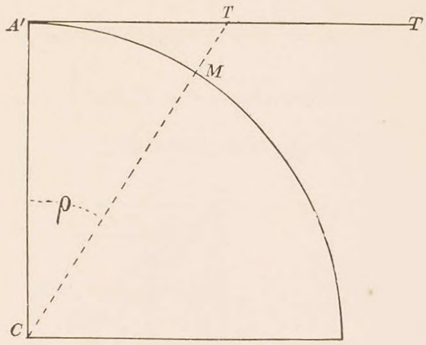

FIG. 17.

Determination of the Axial Elements. For β, measure the arc 100 : 001 with protractor No. 1 For a and c lay off in the fourth quadrant, Fig. 17, ρ of an imaginary face, M, found as described

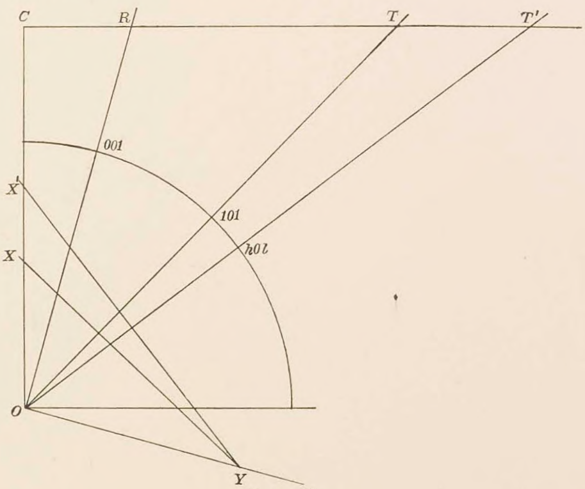

FIG. 18.

above, then if $CA' = b = 1$, the tangent $A'T = c$. In determination of the ratio $\frac{h}{l}$, Fig. 15, if $RT = 1$, $CR = \frac{a}{c}$.

Triclinic System.

Determination of Ratio $\frac{h}{k}$.—Draw in the first quadrant radii at angles φ to ZB, Fig. 19, for planes 110, hk0, etc. Draw a line parallel

to ZA so that $RT =$ unity of the scale line, then $RT' = \frac{h}{k}$ and RT'' $= \frac{h'}{k'}$. ($h'k'o$ corresponding to $h'k'l'$ is on the great circle through C.)

Determination of Ratio $\frac{k}{l}$—Center protractor IV., its diameter coinciding with diameter AA'. Determine on what great circle

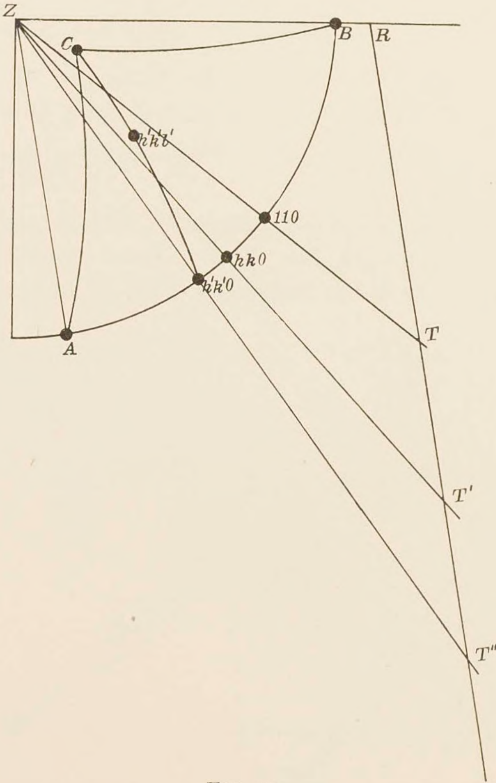

Fig. 19.

measuring in degrees from center) 001 and any face involving a ratio $\frac{k}{l}$ lies. These distances in stereographically projected degree are angles for imaginary planes e, f, g, h, etc., projected on the diameter normal to AA'.

Draw radii, Fig. 20, at angles ρ from the vertical for e, f, g, h. Draw a line parallel to OB so that $RT =$ unity of scale, then $RT' = \frac{k}{l}$; $RT'' = \frac{k'}{l'}$.

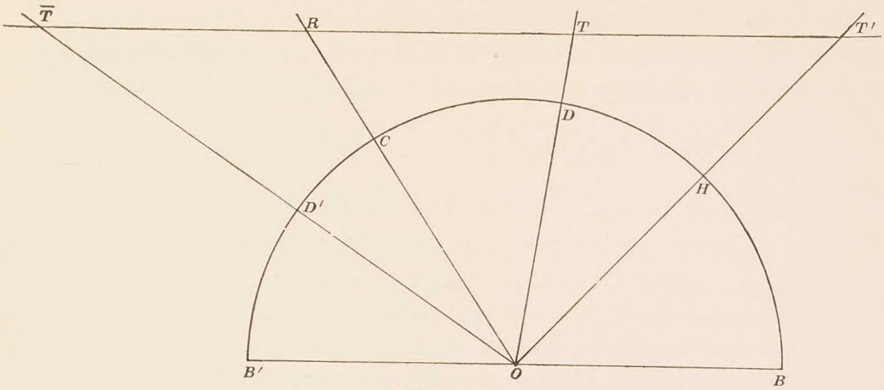

FIG. 20.

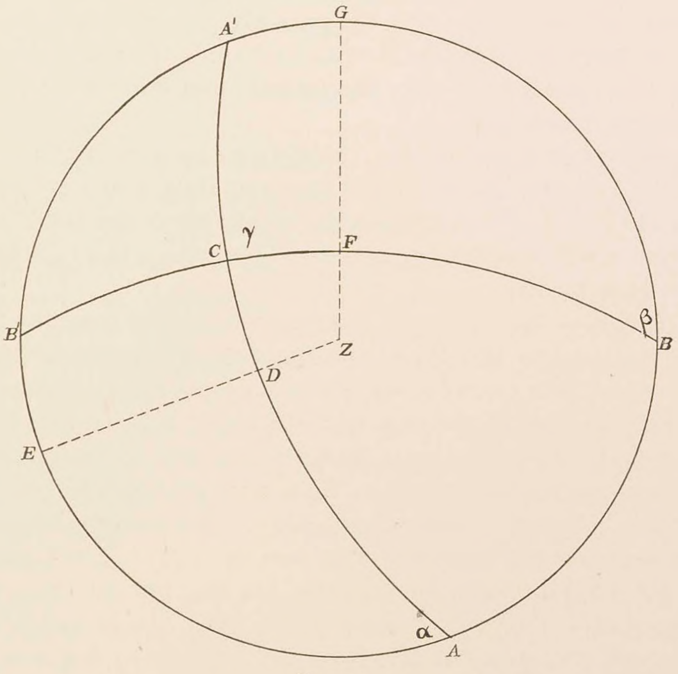

FIG. 21.

Determination of Ratio $\frac{h}{l}$.—As before, but e, f, g, h, are now imaginary planes on a line normal to BB'.

Determination of Axial Elements.—Measure α and β by protractor

No. 1 on radii 90° from desired angles. For instance in Fig. 21 $\alpha = DE$ if $AE = 90°$; $\beta = FG$ if $BG = 90°$, γ is equal to the angle between the tangents to the arcs AA' and BB' at C. In constructions for $\frac{k}{l}$ and $\frac{h}{l}$, Fig, 20, if $RT = 1$, $OR = \frac{b}{c}$ and $\frac{a}{c}$ respectively.

A New Protractor for the Direct Determination of the Indices from the Stereographic Projection.

Plate I. may be cut out and used as a protractor for the determination of indices of faces in the radial zones of the orthometric and hexagonal systems directly from the stereographic projection. The protractor is of a size to be used with a circle of 7 cm. radius, such as the divided circle printed on the sheets designed by Penfield,* and was constructed as follows: Two parallel lines were drawn 7 cm. apart and were divided on a decimal of 7 cm. = unity. On the horizontal line, connecting the parallels at 1, stereographic degrees, the tangents of which are equal to numbers 1 to 10 and tenths from 0.1 to 1.0 were laid off. On several other horizontals similar points were determined.

On the line at 2, for instance, the distance on 1 marked 1 is laid off as ½. On the line at 3 the distance marked 3 on 1 is laid off as 1, and so on. The corresponding points on all horizontal lines were connected, thus forming a system of curves. The unit line is heavier than the others.

To determine the indices of the faces of a radial zone the protractor is placed so that the vertical lines are perpendicular to the zone line and then moved along until the unit line of the protractor falls upon the pole of the unit face, the radial zone always being parallel to the horizontal lines of the protractor. The indices of other faces are read off directly. If the zone is [001 : *okl* : 010] the ratio k/l is indicated. For instance if line 2 of protractor falls upon a pole of a face the indices of that face are 021. If the zone is [001 : *hol* : 100] the ratio h/l is given. In the zone of unit pyramids [001 : *hhl* : 110] h/l is given. The ratio h/k in any *hkl* (or *hko*) is not obtained directly from the protractor, but may be found by locating the corresponding *okl* and *hol* by zone circles (protractor No. IV. of Penfield) and then determining their indices as above.

If the protractor is placed so that its unit line falls upon the pole

* loc. cit.

of the face (hkl) in which $k/l = 1$, the ratio k/l of the other faces of the zone are given. If placed so that the unit line falls upon a pole of a face (hkl) in whick $h/l = 1$, the ratio h/l of other faces results. Or the graphic method given in the article referred to on a preceding page may be employed for the determination of the ratio h/k and the protractor used for the determinations of the ratios k/l and h/l.

In the monoclinic system only the ratio k/l may be determined by the protractor. The protractor is placed on the imaginary orthorhombic zone from center of projection to position 010.

Graphical Determination of Axial Ratio.

In the radial zone [001 : 010] or [0001 : 1120] or the imaginary orthorhombic zone of the monoclinic, the value of the axis $\dot{c}$ is indicated by the reading on the vertical lines, or if the radial zone is [001 : 100] the value a/c is similarly given.

Graphical Determination of Angles.

The protractor may also be used to determine the angle which a dome form with known indices makes with other faces of the zone if the axial ratio is known. For instance to find angle (001 : 021) if $\dot{c} = 1.35$. Determine the distance in stereographic degrees (measured on protractor I.) of 2 line from the left vertical along the horizontal line connecting the 1.35 points. To determine angle [001 : 102] if $a/c = 2.1$, find along the line connecting the 2.1 points on the protractor the distance in stereographic degrees of the ½ line (k/l) from the left vertical line.

In spite of the limitations of the protractor it may prove useful and a fair degree of accuracy is claimed for it. For the determination of faces with complex or very high indices it cannot of course take the place of more accurate graphic determination, but such faces occur rarely.

It may be used in the orthometric and hexagonal systems for the complete determination of a crystal, including indices and axial ratios, from the stereographic projection without calculation or construction.

Example.

Let Fig. 11 (p. 15 in preceding article) represent one quadrant of the projection of an orthorhombic crystal. To determine from the projection the axial ratio and the indices of all faces. Assume

certain faces $o(011)$, $m(110)$ as unit faces. Draw zone circles through C, B and A. The okl planes corresponding to pyramids f, z and τ are located by protractor IV. upon radius CB at (—), o and δ respectively. The hol planes corresponding to f and χ are located at l, z at (—), τ at (—), upon radius ac.

Take the zone [001 : 010]. Place unit line of protractor on o, then a is located at 0.12 (a little over 0.1); (—), at 0.25; ν, at 0.33; φ, at 0.5; ϑ, at 2.0; β, at 3.0. $\dot{c}$ as indicated on verticals of protractor is 1.29, the calculated value being 1.289.

Zone circles through o and m show that 101, which does not occur on the crystal lies at (—). Then using protractor on zone [001 : 100] we find l at 0.25, e at 0.33, d at 0.50, (—) at 2.0. a/c as indicated on vertical is 1.6. $\breve{a}$ may be found by prolonging radius through 110. Then distance $AT = \breve{a} = 0.782$, calculated being 0.785.

Hence the indices of the faces are as given below.

The coördinate angles of some of the forms as found by the protractor together with the calculated values are also given to show the accuracy of the determination of the coördinate angles when the indices and axial elements are known.

		ρ Found.	ρ Calc.
a	018	9° 15′	9° 9′
ι	014	18 5	17 52
ν	013	23 0	23 15
φ	012	32 40	32 48
δ	021	69 0	68 48
β	031	75 20	75 30
l	104		
e	103		
d	102		
f	114		
z	111		
τ	221		
χ	144		
t	121		
s	132		

GRAPHICAL DETERMINATION OF INDICES FROM INTERFACIAL ANGLES.

In spite of the great advantages of measurements by means of the two-circle goniometer and of the convenience of coördinate angles for the purpose of record and comparison it may be found

at times more convenient to make use of the one-circle goniometer and interfacial angles.

There is a convenient method for graphically determining the indices from the interfacial angles. In the orthometric and hexagonal systems, and certain zones of the monoclinic and triclinic the method is exactly the same as that given under the coördinate angles, simple relations existing in these cases between the coördinate and interfacial angles. In other zones of the monoclinic and triclinic a slightly different method is employed.

We may illustrate the principle by one zone of a triclinic crystal, for instance the zone $BC\bar{B}$ (Fig. 21, p. 21).

Let Fig. 20, p. 21, be horizontal projection of zone in question. From B (010) lay off angles BH (010 : $0kl$) BD (010 : 011) BC (010 : 001) BD' (010 : $0\bar{1}1$) BH' (010 : $0\bar{k}l$), etc. (These angles are found from the projection by protractor No. II. of Penfield.) Draw radii through these points. Then the intercepts which the radii make on a line drawn parallel to BB' are in the ratio of the indices. If the line is located so that the distance RT between the radii through C (001) and D (011) is equal to unity of the scale line,* then $RT' = k/l$. Similary if D is $0\bar{1}\bar{1}$, $R\bar{T}$ is 1, and $R\bar{T}' = \bar{k}/l$. For a zone BKB' the distances RT and RT' are in ratio of k/k', if P is $h1l$ then $RT' = k'$. Similarly for zone APD if P is $1kl$, $RT' = h'$. For the zone CPL, if P is $hk1$, $RT' = l'$.

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* Radius of circle is a convenient unit. (Scale No. 4 of Penfield sheets.)